

$$1. A = \{ \text{foo}, \text{bar}, \text{baz} \}$$

$$f: A \rightarrow A$$

(a) Consider $\forall a \in A \quad a, b \in A \quad (a, b) \in R$

$$(a, a) \in R \quad (a, b) \in R \quad (a, b) \in R$$

$$(b, a) \in R \quad (b, c) \in R \Rightarrow (a, c) \in R$$

Symmetric: $A = \{ \text{foo}, \text{bar}, \text{baz} \}$

$$R = \{ (\text{foo}, \text{bar}), (\text{bar}, \text{foo}) \}$$

Transitive: $A = \{ \text{foo}, \text{bar}, \text{baz} \}$

$$R = \{ (\text{foo}, \text{bar}), (\text{bar}, \text{baz}) \}$$

(b) Injective

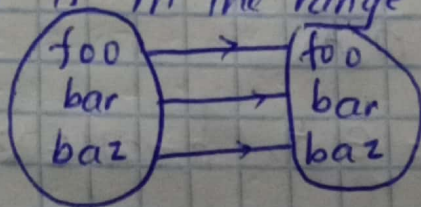
$f: A \rightarrow A$ one-to-one function

When $a_1, a_2 \in A$ with $a_1 \neq a_2$, then $F(a_1) \neq F(a_2)$

If $F(a_1) = F(a_2)$ for $a_1, a_2 \in A$, then it is one to one function

Surjective

$A \rightarrow A$ is in the range of $F(A) = A$



2. (a) $f: A \rightarrow B$

$$R = \{ (a_1, a_2) \in A \times A : (f(a_1), f(a_2)) \in S \}$$

$$\text{Let } a \in A \\ f(a) \in B$$

$$(f(a), f(a)) \in S$$

R is reflexive

(b) Let $a_1, a_2 \in A$ such that $(a_1, a_2) \in R$
as $(a_1, a_2) \in R$

$$(f(a_1), f(a_2)) \in S$$

R is symmetric

(c) Let $a_1, a_2, a_3 \in A$ such that
 $(a_1, a_2) \in R$ and $(a_2, a_3) \in R$

$$\text{As } (a_1, a_2) \in R \Rightarrow (f(a_1), f(a_2)) \in S$$

$$(a_2, a_3) \in R \Rightarrow (f(a_2), f(a_3)) \in S$$

$$(f(a_1), f(a_3)) \in S$$

$$(a_1, a_3) \in R$$

R is transitive

3. (a) (i) $a \in A$ and $A \subseteq B$, then $a \in B$

$\therefore (a, a) \in S$ and $a \in A \rightarrow (a, a) \in A \times A$

$(a, a) \in S$ & $(a, a) \in A \times A$

$(a, a) \in S \cap (A \times A)$

$S \cap (A \times A)$ is reflexive

(ii) Let $a, b \in A$ such that $(a, b) \in S$ and $(a, b) \in S \cap (A \times A)$

$(a, b) \in S$ and $(a, b) \in A$

$a, b \in A$ and $A \subseteq B \rightarrow a, b \in B$

$(a, b) \in A \times A \Rightarrow a \in A, b \in A \rightarrow (b, a) \in A \times A$

$(b, a) \in S$ and $(b, a) \in A \times A$

$(b, a) \in S \cap (A \times A)$

It is symmetric

(iii) $(a, b), (b, c)$ in S and $(a, b), (b, c)$ in $A \times A$

$a, b, c \in A$ and $A \subseteq B \rightarrow a, b, c \in B$

S is an equivalence relation on B

So, $(a, b), (b, c)$ in S

$(a, c) \in S$

$(a, b), (b, c)$ in $A \times A$

a, b, c in $A \rightarrow (a, c) \in A \times A$

$(a, c) \in S$ and $(a, c) \in A \times A$

$(a, c) \in S \cap A \times A$

It is transitive

• $S \cap A \times A$ is an equivalence relation on A

(b) Let $A \subseteq B$, $R = S \cap (A \times A)$
 $[a]_R = [a]_S \cap A$

Let $y \in [a]_R$ and $a \in A$ $[a]_R = \{y \in A : aRy\}$

$aRy \rightarrow (a, y) \in S \cap (A \times A)$

$(a, y) \in S \rightarrow aSy \rightarrow y \in [a]_S$

$(a, y) \in A \times A \rightarrow y \in A$

$y \in [a]_S \cap A$

Let $y \in [a]_S \cap A \Rightarrow y \in [a]_S$ and $y \in A$

$(a, y) \in S$ and $(a, y) \in A \times A$

$(a, y) \in S \cap (A \times A)$

$(a, y) \in R$
 $aRy \rightarrow y \in [a]_R$

$[a]_S \cap A \subseteq [a]_R$

$[a]_S \cap A = [a]_R \rightarrow [a]_R = [a]_S \cap (A \times A)$

$[a]_R \subseteq A$ only and not $A \times A$

$[a]_R = [a]_S \cap A$

4 (a) Since $f(a) = f(a) \forall a \in A, (a, a) \in R_f \forall a \in A$

R_f is reflexive

Since $f(a_1) = f(a_2) \Rightarrow f(a_2) = f(a_1) \forall a_1, a_2 \in A$

R_f is symmetric

Let $(a_1, a_2), (a_2, a_3) \in R_f$ then $f(a_1) = f(a_2)$ and a_2

$f(a_1) = f(a_3) \Rightarrow (a_1, a_3) \in R_f$

R_f is transitive

R_f is an equivalence relation for every $f: A \rightarrow B$

(b) Let $(a_1, a_2) \in E$ then $[a_1] = [a_2]$

$$\Rightarrow q(a_1) = [a_1] = [a_2] = q(a_2) \Rightarrow (a_1, a_2) \in R_q.$$

Thus, $E \subseteq R_q$.

Let $(a_1, a_2) \in R_q$ then $q(a_1) = q(a_2)$

$$[a_1] = [a_2] \Rightarrow (a_1, a_2) \in E$$

Thus, $R_q \subseteq E$

Therefore, $R_q = E$.

$$5. \mathcal{F} = \{B_i : i \in I\}$$

$$I = \{0, 1, 2, 3\}$$

$$i = 0 \quad B_0 = \{(0, 0, 0)\}$$

$$i = 1 \quad B_1 = \{(0, 0, 1), (0, 1, 0), (1, 0, 0)\}$$

$$i = 2 \quad B_2 = \{(0, 1, 1), (1, 0, 1), (1, 1, 0)\}$$

$$i = 3 \quad B_3 = \{(1, 1, 1)\}$$

$$\mathcal{F} = \{B_0, B_1, B_2, B_3\}$$

$$= \{(0, 0, 0), \{(0, 0, 1), (0, 1, 0), (1, 0, 0)\}, \{(0, 1, 1), (1, 0, 1), (1, 1, 0)\}, \{(1, 1, 1)\}\}$$

$\mathcal{F}$ is a partition of B

6 (a) $(a, a) \in E \quad \forall a \in \mathbb{Z}$, E is reflexive

Let $(a, b) \in E$, then $\exists q \in \mathbb{Z}$, $a = b + dq$, then

$$b = a - dq = a + (-q)d, \text{ where } -q \in \mathbb{Z}$$

So, $(b, a) \in E$ E is symmetric

Let $(a, b), (b, c) \in E$, Then $\exists q_1, q_2 \in \mathbb{Z}$ such that
 $a = b + q_1 d$, $b = c + q_2 d$

$$a = b + q_1 d + (c + q_2 d) + q_3 d = c + (q_1 + q_2 + q_3) d$$

$$= c + q_3 d \text{ when } (q_1 + q_2 + q_3) = q_3$$

$(a, c) \in E$ is transitive

E is an equivalence relation.

(b) Let $a_1 \equiv a_2 \pmod{d}$ $b_1 \equiv b_2 \pmod{d}$

$$(a_1, a_2), (b_1, b_2) \in E \quad \exists q_1, q_2 \in \mathbb{Z}$$

$$a_1 = a_2 + q_1 d \text{ and } b_1 = b_2 + q_2 d$$

$$\text{But } a_1 + q_2 = (b_1 + b_2) + (q_1 + q_2) d$$

$$= (b_1 + b_2) + q d, \quad q = q_1 + q_2 \in \mathbb{Z}$$

$$(a_1 + q_2, b_1 + b_2) \in E \quad a_1 + q_2 \equiv b_1 + b_2 \pmod{d}$$

(c) Let $a_1 \equiv a_2 \pmod{d}$ $b_1 \equiv b_2 \pmod{d}$

$$a_1 - a_2 = (b_1 - b_2) + (q_1 - q_2) d$$

$$= (b_1 - b_2) + q' d, \quad q' = q_1 - q_2 \in \mathbb{Z}$$

$$(a_1 - a_2, b_1 - b_2) \in E$$

$$a_1 - a_2 \equiv b_1 - b_2 \pmod{d}$$

7. (a) Since $f(x) = b$ for some $b \in B$
 $\forall x \in A$

$$g \circ f(x) = g(f(x)) = g(b) \quad \forall x \in A$$

$$\exists g(b) \in C \text{ such that } g \circ f(x) = g(b) \quad \forall x \in A$$

$$\text{Let } g(b) = k \in C \Rightarrow g \circ f(x) = k \quad \forall x \in A$$

$g \circ f$ is constant for every function $g: B \rightarrow C$

(b) Since $g \circ f$ is constant

$$\Rightarrow \exists k \in C \text{ subject to } g \circ f(x) = k \quad \forall x \in A$$

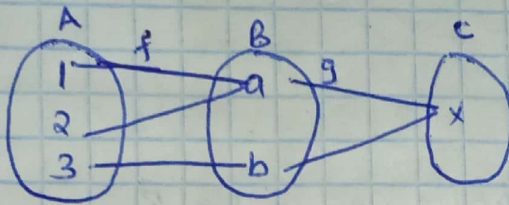
Let g be identity function

$$g \circ f(x) = I \circ f(x) = I(f(x)) = f(x) = k \quad \forall x \in A$$

$\exists k \in B$ subject to $f(x) = k \quad \forall x \in A$

$\Rightarrow f: A \rightarrow B$ is constant

8. (a)

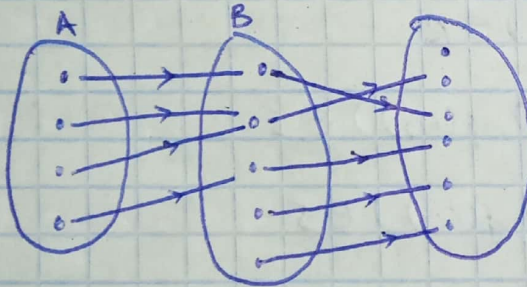


$$g \circ f(x) = g(f(x))$$

$f(x)$ is onto $g(x)$, The domain of $f(x)$ will be range of $g(x)$

and given g is not one-to-one, $g \circ f(x)$ will also not be one-to-one

(b)



If f is not onto, it is possible that for any x , there is not exist any value in range of B

Therefore, $g \circ f$ will have also not any value in Range C .

$\therefore g \circ f$ is not onto